Answer the questions

(1) Find the area of the parallelogram ABCD and the length of the altitude DE in the figure below:

(2) The sides of a triangle are 11 cm, 13 cm and 20 cm. The altitude to the longest side is:

(3) From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 6 cm, 8 cm and 4 cm. Find the area of the triangle.

(4) A rhombus has all its internal angles equal. If one of the diagonals is 7 cm, find the length of other diagonal and the area of the rhombus.

(5) Find the percentage increase in the area of a triangle if each side is increased by Y times.

(6) Find the area of the unshaded region in the figure below:
Ankur makes the kite using two pieces of paper. 1st piece of paper is cut in the shape of square where one diagonal is of the length 30 cm. At one of the vertex of this square a second piece of paper is attached which is of the shape of an equilateral triangle of length to give the shape of a kite. The length of the sides of triangle is a, such that \(a^2 = 8\sqrt{3}\). Find the area of this kite.

Choose correct answer(s) from given choice

(8) If in the figure below \(AB = 16\) cm, \(BC=26\) cm, \(CA = 13\) cm and \(BE = 27\) cm, find the area of the trapezium BDCE.

![Diagram of trapezium BDCE]

(a) 82.94 cm\(^2\)  
(b) 506.86 cm\(^2\)  
(c) 165.88 cm\(^2\)  
(d) 316.79 cm\(^2\)

(9) The adjacent sides of a parallelogram are 8 cm and 11 cm. The ratio of their altitudes is:

(a) 8:19  
(b) 64:121  
(c) 8:11  
(d) 8:5.5

(10) The perimeter of a triangular field is 420 m and the ratio of the sides is 20:15:7. Which of the following is the area of the field in sq m:

(a) 2940  
(b) 84000  
(c) 2800  
(d) 4200

(11) Find the area of a trapezium, if its height is 4 cm and the lengths of parallel sides are 13 cm and 17 cm.

(a) 60 cm\(^2\)  
(b) 884 cm\(^2\)  
(c) 78 cm\(^2\)  
(d) 120 cm\(^2\)

(12) Find the area of a quadrilateral whose sides are 7 cm, 24 cm, 17 cm and 26 cm and the angle between first two sides is a right angle.

(a) 204 cm\(^2\)  
(b) 288 cm\(^2\)  
(c) 850 cm\(^2\)  
(d) 84 cm\(^2\)
(13) If in the figure below AB = 13 cm, BC = 15 cm and CA = 4 cm, find the area of the rectangle BDCE.

![Diagram with dimensions AB = 13 cm, BC = 15 cm, CA = 4 cm]

a. 108 cm²  

b. 24 cm²  

c. 129.6 cm²  

d. 48 cm²

(14) In Heron's formula, \( S = \sqrt{S(S-a)(S-b)(S-c)} \). S is equal to:

a. \( \frac{a + b + c}{abc} \)  

b. \( a + b + c \)  

c. Half of perimeter of the triangle  

d. \( \frac{a \times b \times c}{2} \)

(15) The perimeter of a triangular field is 36 and the ratio of the sides is 8:5:5. The area of the field in sq m is:

a. 128  

b. 48  

c. 1600  

d. 11.313708498985
Step 1

The diagonal AC divides the parallelogram ABCD into two equal triangles, ΔABC and ΔACD. The area of the parallelogram ABCD = 2 × Area(ΔABC).

Step 2

The area of the ΔABC can be calculated using Heron's formula, since all sides of the triangle are known.

\[
S = \frac{(AB + BC + CA)}{2} = \frac{(6 + 25 + 29)}{2} = 30 \text{ cm.}
\]

The area of the ΔABC = \(\sqrt{S(S-AB)(S-BC)(S-CA)}\)

= \(\sqrt{30(30-6)(30-25)(30-29)}\)

= 60 cm\(^2\)

Step 3

The area of the parallelogram ABCD = 2 × Area(ΔABC)

= 2 × 60

= 120 cm\(^2\)

Step 4

The length of the altitude DE = \(\frac{2 \times \text{Area}(\Delta \text{ABC})}{AB}\)

= \(\frac{2 \times 60}{6}\)

= 20 cm.
Step 1
Let’s assume the altitude to the longest side be ‘h’.

Following picture shows the required triangle,

The area of the triangle $\Delta ABC$ can be calculated using Heron’s formula, since all sides of the triangles are known.

\[
S = \frac{(AB + BC + CA)}{2} \\
= \frac{(20 + 13 + 11)}{2} \\
= 22 \text{ cm.}
\]

The area of the $\Delta ABC = \sqrt{S(S - AB)(S - BC)(S - CA)}$

\[
= \sqrt{22(22 - 20)(22 - 13)(22 - 11)} \\
= 66 \text{ cm}^2
\]

Step 2

The altitude to the longest side = \(\frac{2 \times (\text{The area of the } \Delta ABC)}{\text{Base 'AB'}}\)

\[
= \frac{2 \times 66}{20} \\
= 6.6 \text{ cm.}
\]
**Step 1**

Following figure shows the required triangle,

Let's assume the sides of the equilateral triangle ΔABC be \(x\).

The area of the triangle ΔABC can be calculated using Heron's formula, since all sides of the triangles are known.

\[
S = \frac{(AB + BC + CA)}{2} = \frac{(x + x + x)}{2} = \frac{3x}{2} \text{ cm.}
\]

The area of the ΔABC = \(\sqrt{S(S - AB)(S - BC)(S - CA)}\)

\[
= \sqrt{\left(\frac{3x}{2}\right)\left(\frac{x}{2}\right)^3}
\]

\[
= \sqrt{\frac{3(x/2)^4}{4}}
\]

\[
= \frac{\sqrt{3}}{4} (x)^2 \quad (1)
\]

**Step 2**

The area of the triangle AOB = \(\frac{AB \times OP}{2}\)

\[
= \frac{x \times 8}{2} = \frac{8x}{2}
\]

**Step 3**
Similarly, the area of the triangle $\triangle BOC = \frac{6x}{2}$

and the area of the triangle $\triangle AOC = \frac{4x}{2}$.

**Step 4**

The area of the triangle $\triangle ABC = \text{Area}(\triangle AOB) + \text{Area}(\triangle BOC) + \text{Area}(\triangle AOC)$

$$= \frac{8x}{2} + \frac{6x}{2} + \frac{4x}{2}$$

$$= \frac{18x}{2}$$

----- (2)

**Step 5**

By comparing equation (1) and (2), we get,

$$\frac{\sqrt{3}}{4} (x)^2 = \frac{18x}{2}$$

$$\Rightarrow x = \frac{36}{\sqrt{3}}$$

**Step 6**

Now, $\text{Area}(\triangle ABC) = \frac{\sqrt{3}}{4} (x)^2$

$$= \frac{\sqrt{3}}{4} \left( \frac{36}{\sqrt{3}} \right)^2$$

$$= \frac{324}{\sqrt{3}} = 108 \sqrt{3} \text{ cm}^2$$

**Step 7**

Hence, the area of the triangle is $108\sqrt{3} \text{ cm}^2$.

(4) 7 cm, 24.5 cm$^2$

The key thing to note is that all the internal angles of a rhombus add up to 360°. So if all the internal angles are equal, then one internal angle is $360° \div 4 = 90°$. This is a square with all sides equal. So the other diagonal is also 7 cm in length. The area of a rhombus/square is half the product of the diagonals.

Area = $(7 \times 7) \div 2 = 49 \div 2 = 24.5 \text{ cm}^2$
Step 1
Consider a triangle QRS with sides a, b and c. Let \( S = \frac{a+b+c}{2} \)
Area of triangle QRS = \( A_1 = \sqrt{S(S-a)(S-b)(S-c)} \)

Step 2
Increasing the side of each side by \( Y \) times, we get a new triangle XYZ
XYZ has sides \( Ya, Yb \) and \( Yc \)
By Heron’s formula
Area of new triangle = \( \sqrt{S_1(S_1-Ya)(S_1-Yb)(S_1-Yc)} \)
Where \( S_1 = \frac{Ya + Yb + Yc}{2} = Y \times \frac{a+b+c}{2} = MS \)
Area of XYZ = \( \sqrt{Y^4 S(S-a)(S-b)(S-c)} \)
= \( Y^2 \times A_1 \)
This means the area increases by \( 100Y^2 \% \)
If we look at the figure carefully, we notice that, the area of the unshaded region = The area of the triangle $\triangle ABC$ - The area of the triangle $\triangle ACD$.

**Step 2**

The area of the triangle $\triangle ABC$ can be calculated using Heron's formula, since all sides of the triangle are known.

\[ S = \frac{(AB + BC + CA)}{2} \]
\[ = \frac{(28 + 17 + 39)}{2} \]
\[ = \frac{42}{2} \]
\[ = 21 \text{ m}^2 \]

The area of the $\triangle ABC$ = $\sqrt{S(S - AB)(S - BC)(S - CA)}$
\[ = \sqrt{42(42 - 28)(42 - 17)(42 - 39)} \]
\[ = 210 \text{ m}^2 \]

**Step 3**

Similarly, the area of the triangle $\triangle ACD$ can be calculated using Heron's formula.

\[ S = \frac{(AC + CD + DA)}{2} \]
\[ = \frac{(39 + 16 + 25)}{2} \]
\[ = \frac{40}{2} \]
\[ = 20 \text{ m}^2 \]

The area of the $\triangle ACD$ = $\sqrt{S(S - AC)(S - CD)(S - DA)}$
\[ = \sqrt{40(40 - 39)(40 - 16)(40 - 25)} \]
\[ = 120 \text{ m}^2 \]

**Step 4**

Thus, the area of the unshaded region = Area($\triangle ABC$) - Area($\triangle ACD$)
\[ = 210 - 120 \]
\[ = 90 \text{ m}^2 \]
Step 1
Following figure shows the kite, made by two pieces of paper,

Step 2
Now, we can see that, this kite consists of a square ABCD and a equilateral triangle BEF. The area of the equilateral triangle ΔBEF can be calculated using Heron’s formula for equilateral triangle,
\[
\text{Area} = \frac{\sqrt{3} a^2}{4}
\]
\[
= \frac{\sqrt{3} (8\sqrt{3})}{4}
\]
\[
= 6 \text{ cm}^2
\]

Step 3
The diagonal of the square = 30 cm.
The area of the square ABCD = \( \frac{1}{2} \times (30)^2 = 450 \text{ cm}^2 \).

Step 4
Thus, the area of the kite = Area(ABCD) + Area(BEF)
= 6 + 450
= 456 \text{ cm}^2.

(8) d. 316.79 \text{cm}^2

(9) c. 8:11

The area of a parallelogram is base \( x \) height, where base refers to the side, and height is indicated by the altitude
\[
\text{Area} = b_1 \times h_1 = b_2 \times h_2
\]
Here, let’s take \( b_1 = 8 \), and \( b_2 = 11 \)
\[
8 \times h_1 = 11 \times h_2 \text{ Taking the ration we get } h_2:h_1 = 8:11
\]
Step 1
Since we know the perimeter, we can use Heron's formula to help us compute the area.
The formula states that the area of a triangle with sides a, b and c, and perimeter $2S = \sqrt{S(S-a)(S-b)(S-c)}$

Step 2
Let us assume the 3 sides are of length $a=20x$, $b=15x$ and $c=7x$ (we know this because the ratio of the sides is given as 20:15:7)

Step 3
We also know that $a+b+c = 420$.

$20x + 15x + 7x = 420$

$(20 + 15 + 7)x = 420$

$42x = 420$

$x = \frac{420}{42} = 10$

Step 4
From this we see that $a = 200$, $b = 150$ and $c=70$. Also $S=210$

Step 5
Putting these values into Heron's formula,

$\text{Area} = \sqrt{210(210-200)(210-150)(210-70)} = \sqrt{17640000}$

Solving, we find the area = 4200
(11) a. 60 cm²

Step 1
The following picture shows the trapezium ABCD,

![Trapezium ABCD diagram](image)

Step 2
According to the question, the height of the trapezium ABCD = 4 cm

The area of the trapezium ABCD = The height of the trapezium ABCD × \( \frac{AB + CD}{2} \)

\[
= 4 \times \frac{17 + 13}{2}
\]

\[
= 4 \times \frac{30}{2}
\]

\[
= 60 \text{ cm}^2
\]

Step 3
Thus, the area of the trapezium is **60 cm²**.
Step 1
Let's ABCD is the quadrilateral with AB = 7 cm, BC = 24 cm, CD = 17 cm, DA = 26 cm, and angle $\angle ABC = 90^\circ$, as shown in the following figure.

Step 2
Let's draw the diagonal AC in the quadrilateral ABCD,

The area of the right triangle ABC = $\frac{1}{2} \times AB \times BC = \frac{1}{2} (7) (24) = 84 \text{ cm}^2$

Step 3
$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(7^2 + 24^2)} = 25 \text{ cm}$

Step 4
The area of the triangle ACD can be calculated using Heron's formula.
$S = (CD + DA + AC)/2 = (17 + 26 + 25)/2 = 34 \text{ cm}$
The area of the triangle ACD = $\sqrt{[ S (S-CD) (S-DA) (S-AC) ]}$
= $\sqrt{[ 34 (34-17) (34-26) (34-25) ]}$
= 204 \text{ cm}^2

Step 5
The area of the quadrilateral ABCD = Area(ABC) + Area(ACD) = 84 + 204 = 288 \text{ cm}^2
Step 1

The area of the triangle ABC can be calculated using Heron’s formula, since all sides of the triangle are known.

\[ S = (4 + 13 + 15)/2 = 16 \text{ cm}. \]

The area of the \( \triangle ABC \) is

\[ \sqrt{(S - AB) (S - BC) (S - CA)} \]

\[ = \sqrt{16(16 - 13) (16 - 15) (16 - 4)} \]

\[ = 24 \text{ cm}^2 \]

Step 2

The height (BD) of the \( \triangle ABC \) is \( \frac{2 \times \text{Area} \( \triangle ABC \)}{AC} \)

\[ = \frac{2 \times 24}{4} \]

\[ = 12 \text{ cm} \]

Step 3

In right angled \( \triangle BDC \), \( DC^2 = BC^2 - BD^2 \)

\[ \Rightarrow DC = \sqrt{BC^2 - BD^2} \]

\[ = \sqrt{(15)^2 - (12)^2} \]

\[ = 9 \text{ cm} \]

The area of the rectangle BDCE = BD \times DC

\[ = 12 \times 9 \]

\[ = 108 \text{ cm}^2 \]
(14) **c. Half of perimeter of the triangle**

**Step 1**
In Heron's formula, \( S \) represents the **semi-perimeter** of the triangle.

**Step 2**
Semi-perimeter is defined as the half of perimeter of the triangle.

(15) **b. 48**

**Step 1**
Since we know the perimeter, we can use Heron's formula to help us compute the area. The formula states that the area of a triangle with sides \( a, b \) and \( c \), and perimeter \( 2S = \sqrt{S(S-a)(S-b)(S-c)} \).

**Step 2**
Let us assume the 3 sides are of length \( a=8x, \) \( b=5x \) and \( c=5x \) (we know this because the ratio of the sides is given as 8:5:5).

**Step 3**
We also know that \( a+b+c = 36 \).

\[
= 36 = 36 \\
(8 + 5 + 5)x = 36 \\
18x = 36 \\
x = \frac{36}{18} = 2
\]

**Step 4**
From this we see that \( a = 16 \) m, \( b = 10 \) m and \( c=10 \) m. Also \( S=18 \).

**Step 5**
Putting these values into Heron's formula,

\[
Area = \sqrt{18(18-16)(18-10)(18-10)}
\]

Area = 48 m\(^2\)