Class 9

Circles

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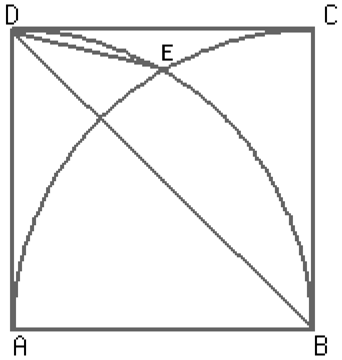
Choose correct answer(s) from the given choices

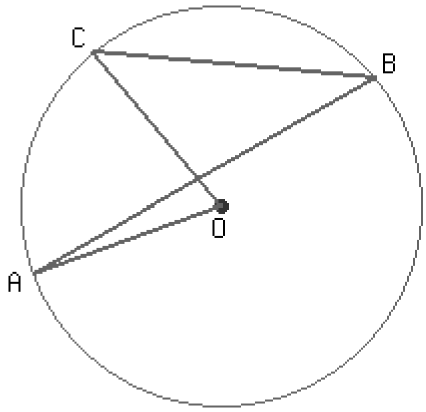
(1) ABCD is a square. Two arcs are drawn with A and B as centers, and radius equal to the side of square. If arcs intersects at point E, find the angle $\angle$BDE.



a. 25° **b.** 30°

c. 35° **d.** 27.5°

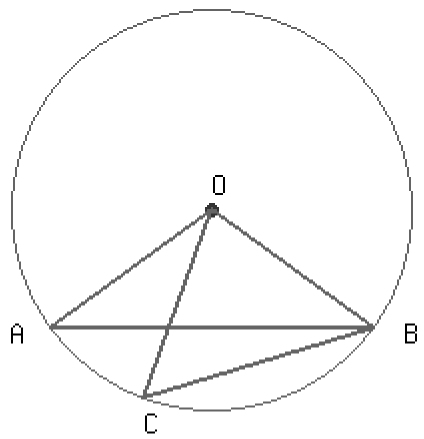
(2) If O is center of the circle and $\angle$AOC = 70°, the value of $\angle$ABC is:



a. 40° **b.** 30°

c. 35° **d.** 25°

(3) If ∠OAB = 36° and ∠OCB = 53°, find ∠BOC.

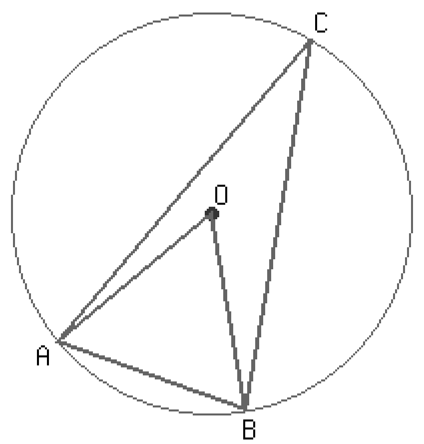


a. 69°
b. 64°
c. 79°
d. 74°

(4) Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A(or B) intersecting the circles at P and Q. Find the ratio PQ:OO′.

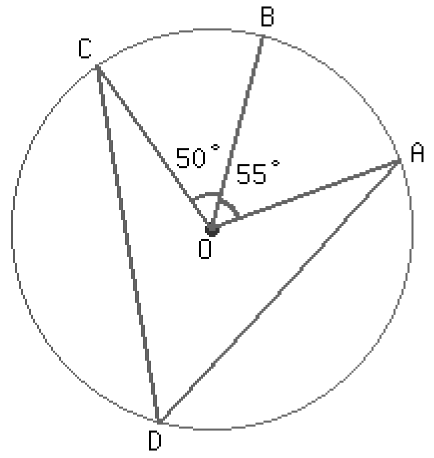
a. 3:2
b. 2:1
c. 1:2
d. 1:1

(5) If O is center of the circle and ∠OAB = 60°, the measure of ∠ACB is:



a. 25°
b. 30°
c. 20°
d. 35°

(6) If O is center of the circle, find angle ∠ADC.



a. 52.5°

b. 55°

c. 50°

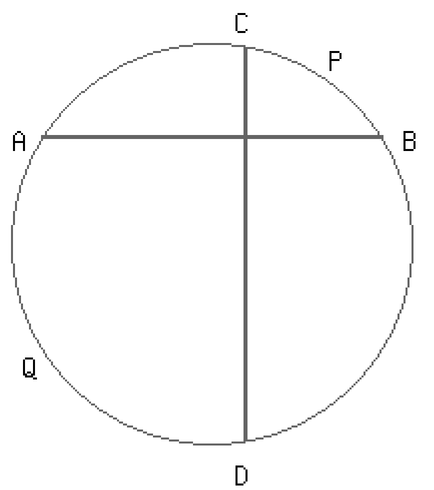
d. 47.5°

Fill in the blanks

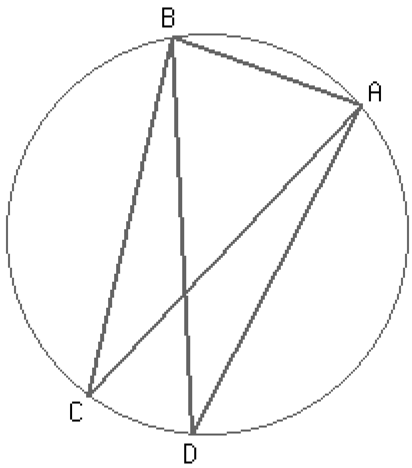
(7) AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 16 cm, the perpendicular distance of AB from the centre of the circle is ______ cm.

Answer the questions

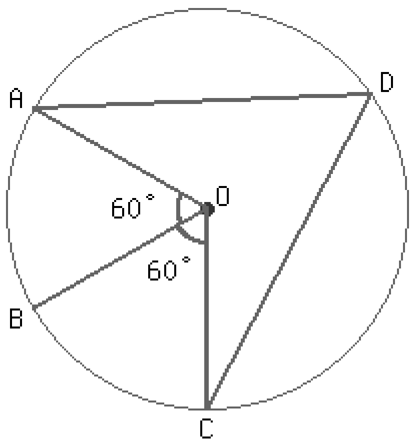
(8) The chords AB and CD of a circle are perpendicular to each other. If radius of the circle is 35 cm and length of the arc BPC is 30 cm, find the length of arc AQD. (Assume $\pi = \frac{22}{7}$)



(9) If ∠DAB = 83° and ∠ABD = 67°, find the value of ∠ACB.



(10) If O is center of the circle, find angle ∠ADC.



Solutions

(1) **b.** 30°

Step 1

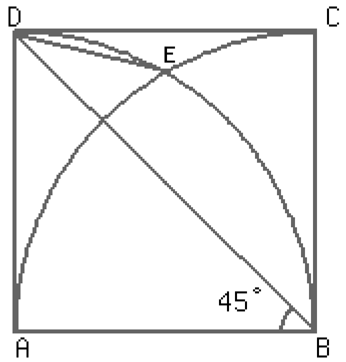
We are given, ABCD is a square with a diagonal BD.

We know,

- All the angles of a square is equal to 90°.
- The diagonal of a square bisects its angles.

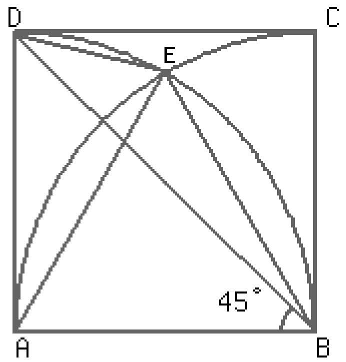
So, ∠B = 90°

and ∠ABD = 45° (BD is a diagonal)



Step 2

Now, join A and B to E.



We know,

AE is the radius of the circle with center A.

BE is the radius of the circle with center B.

AB is the side of the square.

We are given that radius is equal to the side of the square.

So, ΔABE is an equilateral triangle

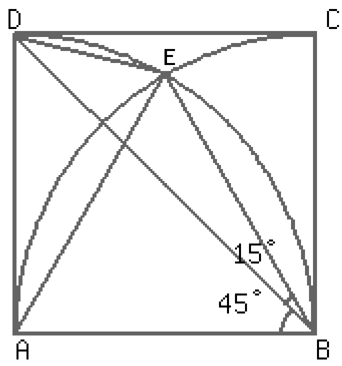
⇒ Each angle of ΔABE is 60°. **(All the angles of an equilateral triangle are 60°.)**

⇒ABE = 60°

Step 3

We can see,

∠DBE = ∠ABE − ∠ABD = 60° − 45° = 15°



Step 4

As, ∠A = 90°

Reflex ∠A = 360° − 90° = 270° **(Angle at a point is 360°)**

Now, ∠DEB = 135° **(The angle subtended by a chord at the center of the circle is double the angle subtended by the same chord at the circumference.)**

Step 5

Consider ΔDEB:

∠DEB = 135°

∠DBE = 15°

∠DEB + ∠DBE + ∠BDE = 180° **(Sum of angles of a triangle are 180°)**

⇒ 135° + 15° + ∠BDE = 180°

⇒ ∠BDE = 180° − (135° + 15°)

⇒ ∠BDE = 180° − (150°)

⇒ ∠BDE = 30°

(2) **c.** 35°

Step 1

The key point to note here is that AC is a chord of the circle.

Step 2

B is a point on the circumference, and O is the center.

Step 3

We also know that the angle subtended by a chord at the centre of a circle is double the angle subtended by the same chord at the circumference of the circle.

Step 4

The angle subtended by the chord AC at the center is 70°. So, the angle subtended by the chord AC at the point on the circumference is 35°.

Step 5

Hence, $\angle ABC = 35°$

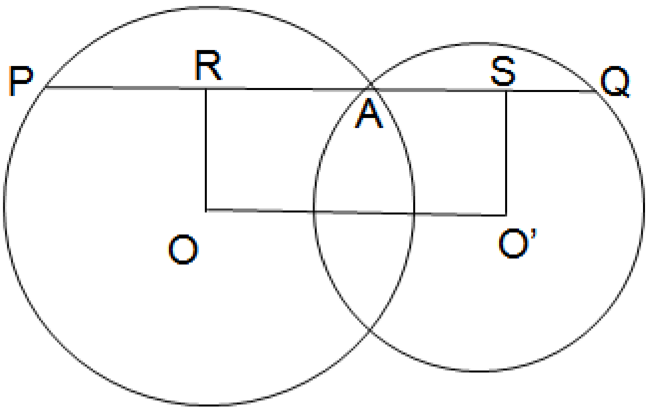
(3) **d.** 74°

In ΔOCB, we see that OC = OB (radius of a circle).
⇒ $\angle OCB = \angle OBC$ (angle opposite to equal sides are equal)
Also, $\angle BOC + \angle OCB + \angle OBC = 180°$ (angle sum property)
So, $\angle BOC = 180° - (\angle OCB + \angle OBC)$.
⇒ $\angle BOC = 180° - 2 \times 53° = 74°$

(4) **b.** 2:1

Step 1

Consider the image below:



We see that the line drawn at point O perpendicular to OO′ meets the line PQ at a point R.
A similar line drawn at O′ meets PQ at a point S.
Now, PA is a chord, and OR is a line perpendicular to it drawn from the centre of the circle.
We know that a perpendicular drawn from the centre of a circle to a chord bisects the chord.
Therefore, OR bisects PA, i.e. PR = RA or PA = 2RA.

Step 2

Similarly, AQ = 2AS.
We know, PQ = PA + AQ
⇒ PQ = 2RA + 2AS
⇒ PQ = 2(RA + AS)
⇒ PQ = 2OO′ (RA + AS is the same as OO′)

Step 3

The ratio of PQ:OO′ is 2:1.

(5) **b.** 30°

Step 1

The key point to note here is that AB is a chord of the circle,C is a point on the circumference, and O is the center.

Step 2

Since, OA and OB are the radius of the circle, OA = OB.
Hence, ΔOAB is an isosceles triangle.
⇒ ∠OAB = ∠OBA
In ΔAOC using angle sum property. We have,
∠OAB + ∠OBA + ∠AOB = 180°
In this case, 2 × (∠OAB) + ∠AOB = 180°
or, ∠AOB = 180° - 2 × (∠OAB)

Step 3

We also know that the angle subtended by a chord at the centre of a circle is double the angle subtended by the same chord at its circumference.
This means ∠AOB = 2 × ∠ACB

Step 4

So, we have 2 equations:
1) ∠AOB = 180° - 2 × (∠OAB)
2) ∠AOB = 2 × ∠ACB

Step 5

Equating both the equations and subsituting ∠OAB = 60°
We get, ∠ACB = 30°.

(6) **a.** 52.5°

Step 1

We see in the image that AC is a chord of the circle.

Step 2

The angle subtended by the chord to the center is twice the angle subtended to a point on the circumference (on the same side as the center).

Step 3

The total angle subtended to the center by AC = 55 + 50 = 105.

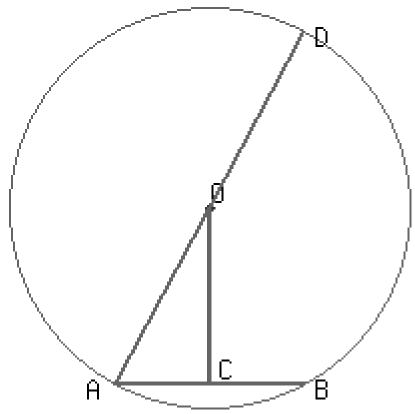
Step 4

Therefore, the angle subtended to D which lies in the circumference = 105 ÷ 2 = 52.5°.

(7) 15

Step 1

Let us draw the figure as follows:



We are told that the diameter AD = 34 cm and the length of the chord AB = 16 cm.
We are asked to find the perpendicular distance of AB from the center O.
This is the length of the line segment OC.

Step 2

We can see that OAC is a right-angled triangle.
We also know that OA is the radius, i.e half of the diameter AD.
Therefore, AO = 17 cm.

Step 3

We know that the perpendicular to the chord from the center i.e. OC, divides the chord into two equal parts.

Therefore, AC = CB = $\frac{AB}{2} = \frac{16}{2}$ = 8 cm

Step 4

Since, OAC is a right angle triangle, $OC^2 + AC^2 = OA^2$

$\Rightarrow OC^2 + 8^2 = 17^2$

$\Rightarrow OC^2 = 289 - 64$

$\Rightarrow OC^2 = 225$

$\Rightarrow$ OC = 15 cm

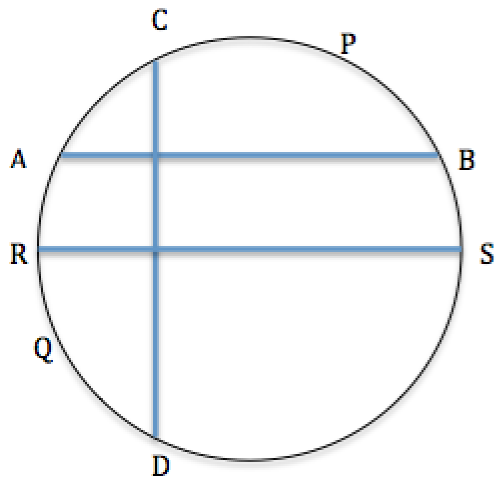
Step 5

Therefore, perpendicular distance of AB from the centre of the circle is **15 cm**.

(8) 80 cm

Step 1

Consider the figure given below:



We have drawn a diameter RS parallel to AB.

Step 2

Now, RS is a diameter perpendicular to CD (As, RS is parallel to AB and AB is perpendicular to CD).
We know, when a diameter is perpendicular to a chord it bisects the chord and its arc.
Therefore, RS bisects arc CD.
So, Arc RC = Arc RD
Also, as AB // RS. We have, Arc AR = Arc BS

Step 3

Since, RS is a diameter. Arc RS cover a semi circle.
Arc RS = Arc RC + Arc CS = Arc RD + Arc CS (As, Arc RC = Arc RD)
= Arc RD + Arc CS - Arc AR + Arc AR (Adding and subtracting Arc AR)
= Arc RD + Arc CS - Arc BS + Arc AR (As, Arc AR = Arc BS)
= (Arc RD + Arc AR) + (Arc CS - Arc BS)
= Arc AQD + Arc BPC

Step 4

We know length of arc BPC is 30 cm, and need to find length of arc AQD.
From the previous analysis, we know that arc BPC and arc AQD cover a semi circle, so the total length of the two arcs is half the circumference.

Circumference of the circle = $2 \pi r = 2 \times \frac{22}{7} \times 35$ cm = 220 cm

Length of arc BPC + Length of arc AQD = $\frac{1}{2} \times 220$ cm = 110 cm

Length of arc AQD = 110 cm - Length of arc BPC = 110 cm - 30 cm = 80 cm.

(9) 30°

Step 1

The angle subtended by a chord at two points on the circumference of a circle are equal, if the two points are on the same side of the chord.

Step 2

This means that, ∠ACB = ∠BDA.

Step 3

Using angle sum property in ΔABD. We have,
∠DAB + ∠ABD + ∠BDA = 180°.
Subsituting the value of ∠DAB and ∠ABD in the above equation. We have,
83° + 67° + ∠BDA = 180°.
⇒ ∠BDA = 180° - (83° + 67°)
⇒ ∠BDA = 30°

Step 4

By step 2, ∠ACB = ∠BDA = 30°

(10) 60°

Step 1

We see in the image that AC is a chord of the circle.

Step 2

The angle subtended by the chord to the center is twice the angle subtended to a point on the circumference (on the same side as the center).

Step 3

The total angle subtended to the center by AC = 60 + 60 = 120.

Step 4

Therefore, the angle subtended to D which lies in the circumference = 120 ÷ 2 = 60°.